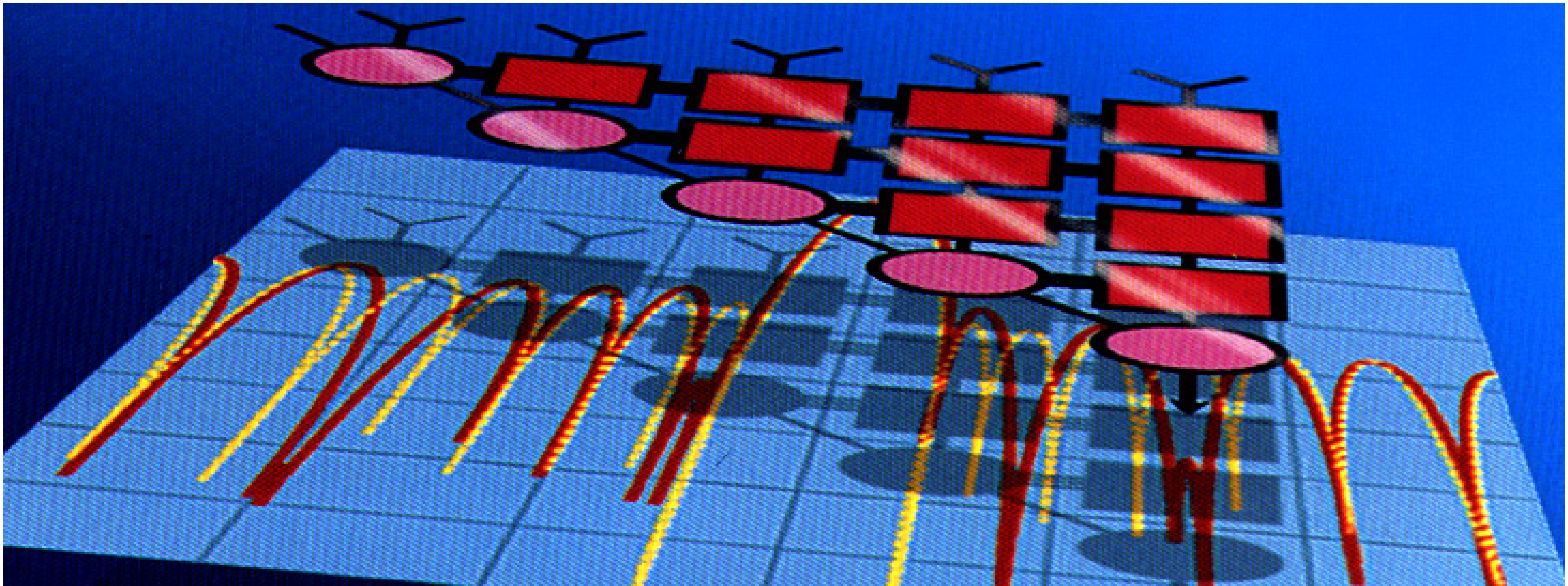


Developments in Sensor Array Signal Processing

John G McWhirter FRS FREng

QinetiQ

Overview of Talk

- Sensor array signal processing
  - historical perspective and overview
- Recent developments and current trends
  - from ABF to BSS
  - from 2nd order statistics to HOS
  - convergence with artificial neural networks
- Current research and future challenges
  - Convolutive mixtures
  - Semi-blind signal separation

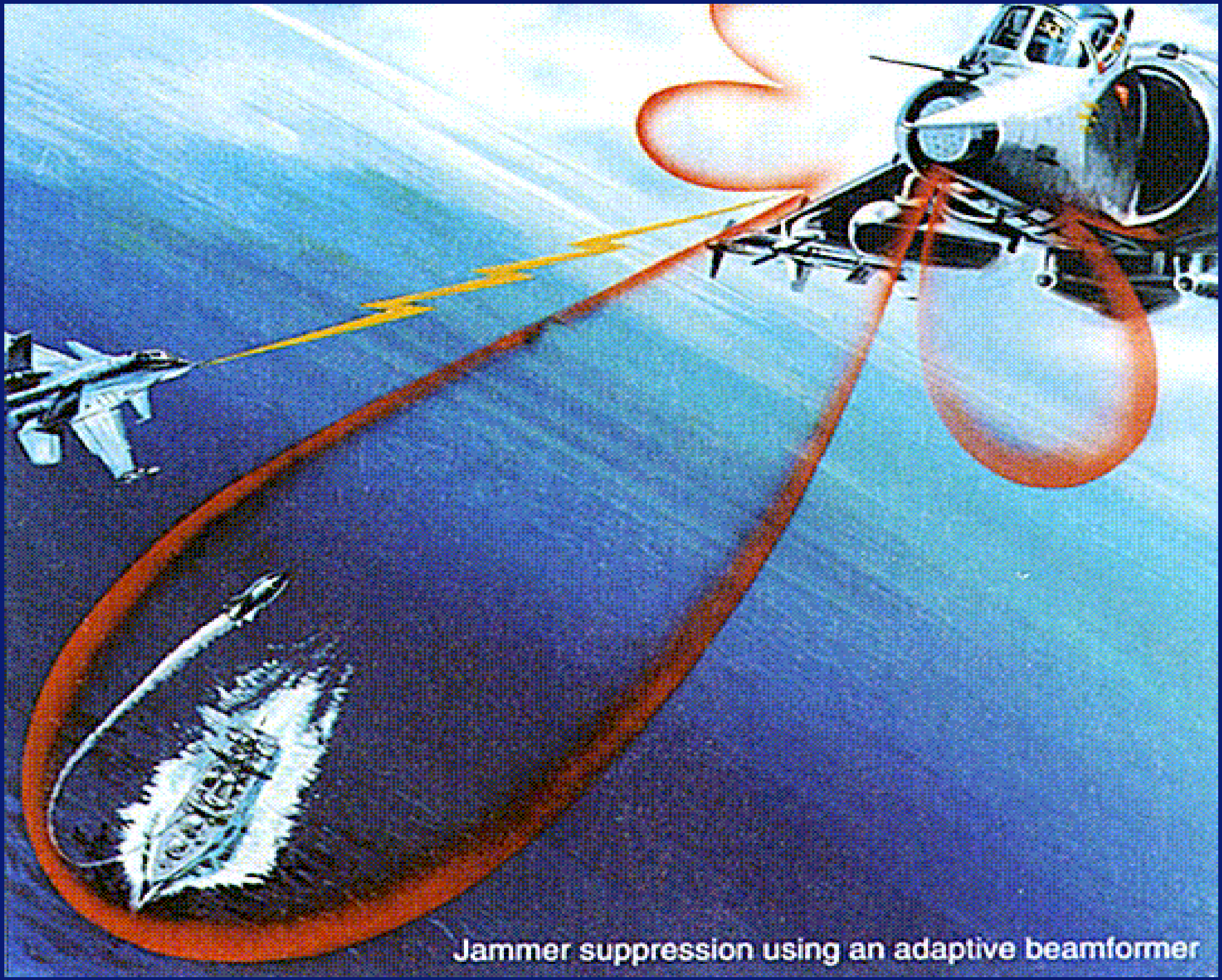
QinetiQ

Sensor Array Signal Processing

- Techniques have recently "come of age"
  - Enabled by the digital processing revolution
  - Impressive research results

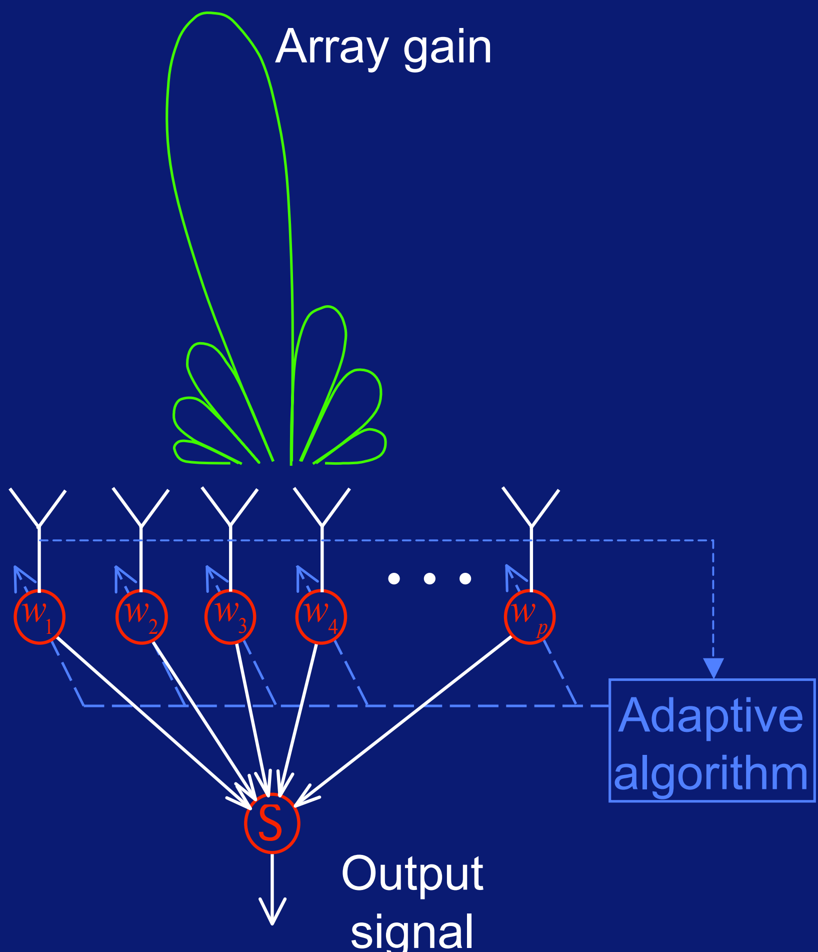
- Wide range of application areas
  - Key to improving mobile telephone systems
  - Could revolutionise design of future radars
  - Medical diagnostic techniques (ECG, EEG)

Adaptive Null Steering



QinetiQ

Adaptive Beamforming



- Complex weights (represent phase and amplitude)
- Output signal

$$e(t) = \mathbf{w}^H \mathbf{x}(t)$$

- Minimise output power subject to look-direction constraint

$$\mathbf{w}^H \mathbf{c}(\theta) = \mu$$

Least Squares Solution

- Minimise

$$E(n) = \sum_{t=1}^{n} |e(t)|^2 = \mathbf{w}^H \mathbf{M}(n) \mathbf{w}$$

  - subject to

$$\mathbf{w}^H \mathbf{c}(\theta) = \mu$$

- Least squares solution (Gauss normal equations)

$$\mathbf{M}(n)\mathbf{w}(n,\theta) = \lambda \mathbf{c}(\theta)$$

  - where

$$M_{ij}(n) = \sum_{t=1}^{n} x_i(t) x_j^*(t)$$

QinetiQ

LMS Algorithm

- Minimise

$$\mathrm{E}\{|e(t)|^2\}$$

  - where

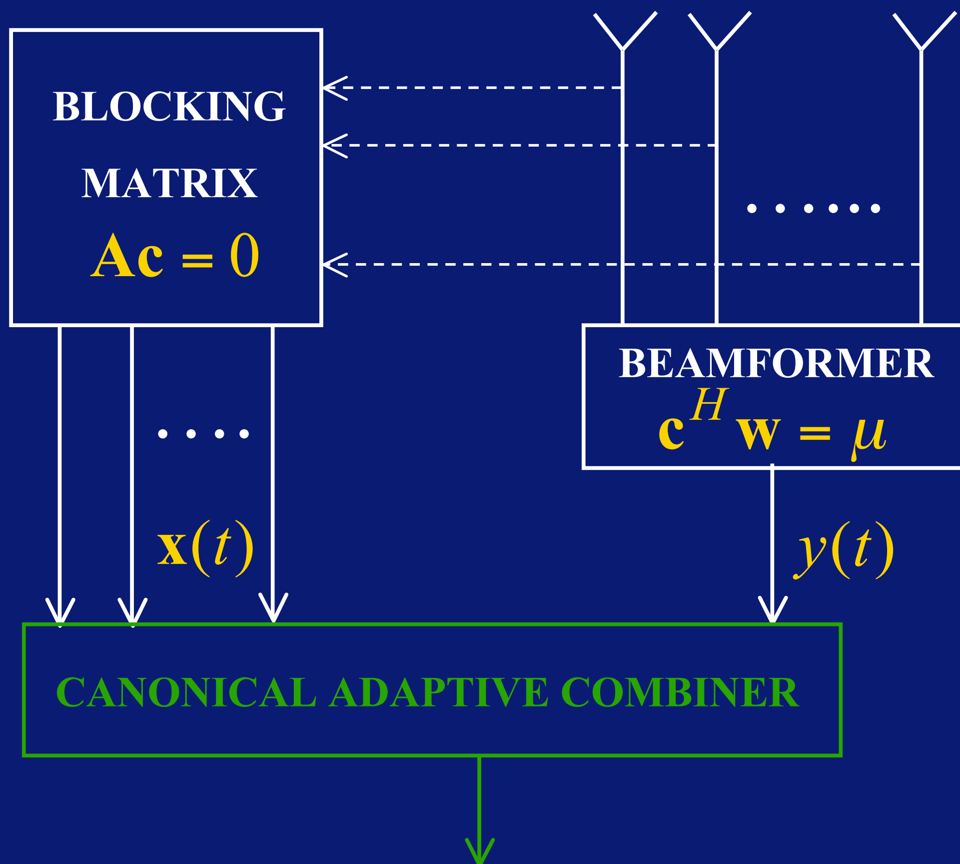
$$e(t) = \mathbf{w}^H \mathbf{x}(t) + y(t)$$

- Stochastic gradient update

$$\mathbf{w}(t+1) = \mathbf{w}(t) - \mu e^*(t)\mathbf{x}(t)$$

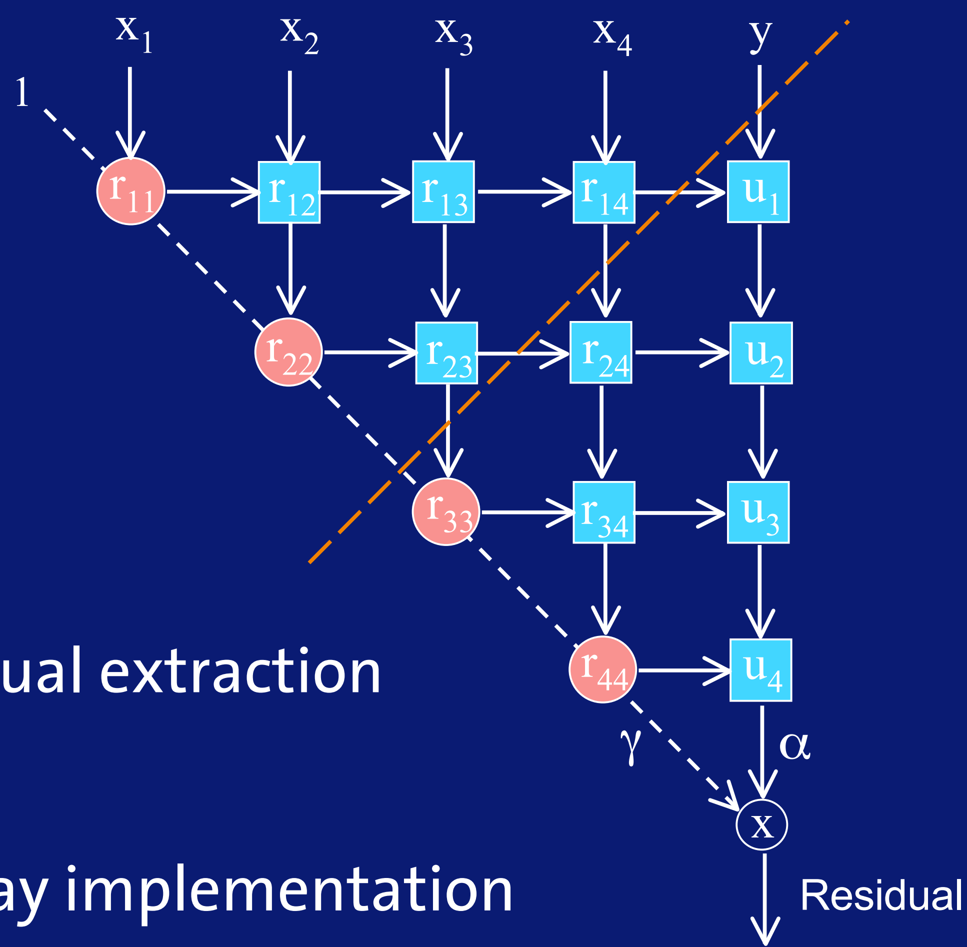
- Minimal computation
- Can be slow to converge

Canonical Problem and GSLC



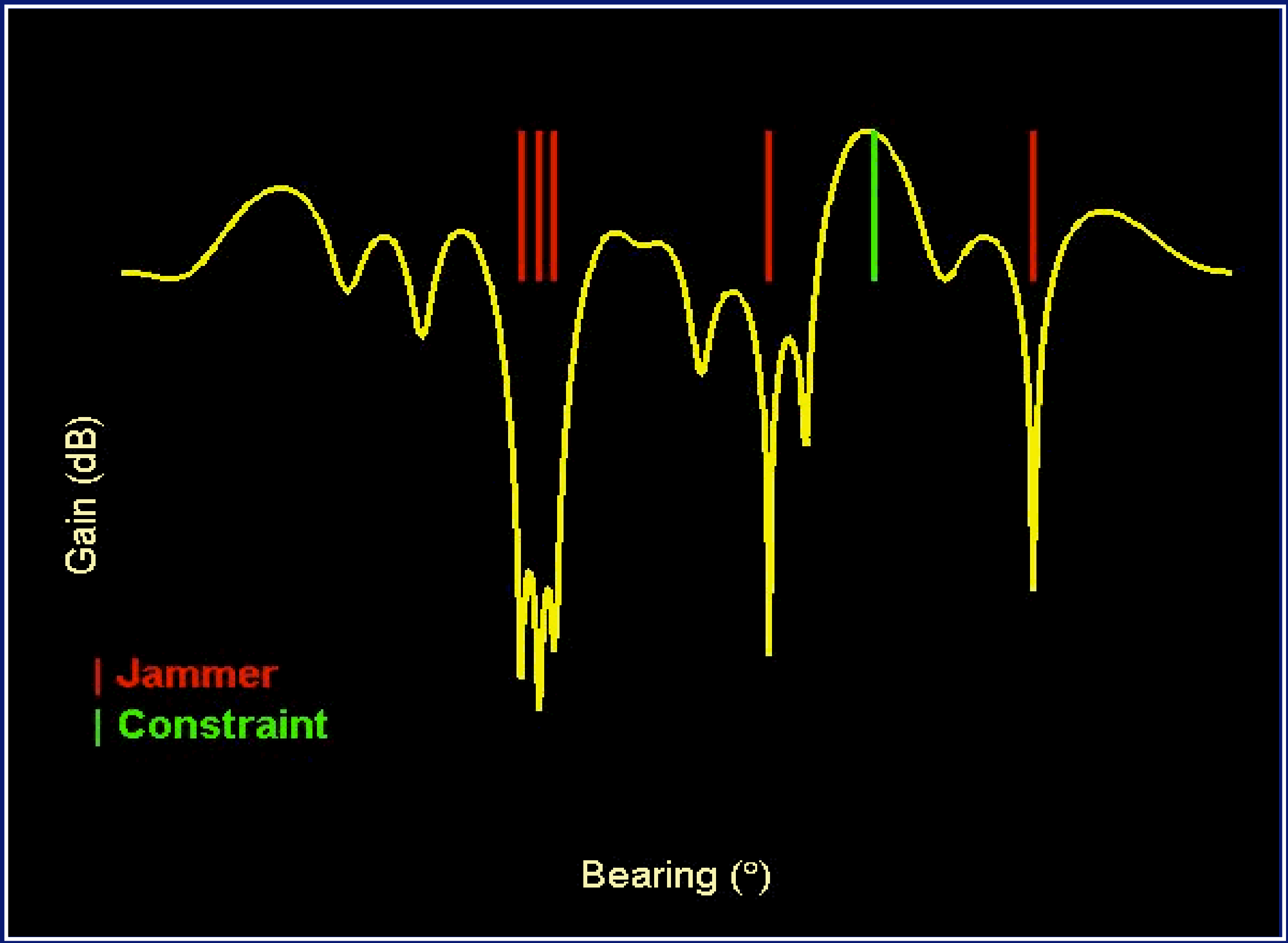
$$e(t) = \mathbf{w}^H \mathbf{x}(t) + y(t)$$

QRD Processor Array



- Direct residual extraction
- Systolic array implementation

Unstabilised Beam Pattern



QinetiQ

Penalty Function Method

- Penalty function

$$\overline{E} = \int_{-\pi/2}^{+\pi/2} h(\theta)\left|(\mathbf{w}-\mathbf{w}_q)^H \mathbf{c}(\theta)\right|^2 d\theta$$

$$= (\mathbf{w}-\mathbf{w}_q)^H \mathbf{Z}(\mathbf{w}-\mathbf{w}_q)$$

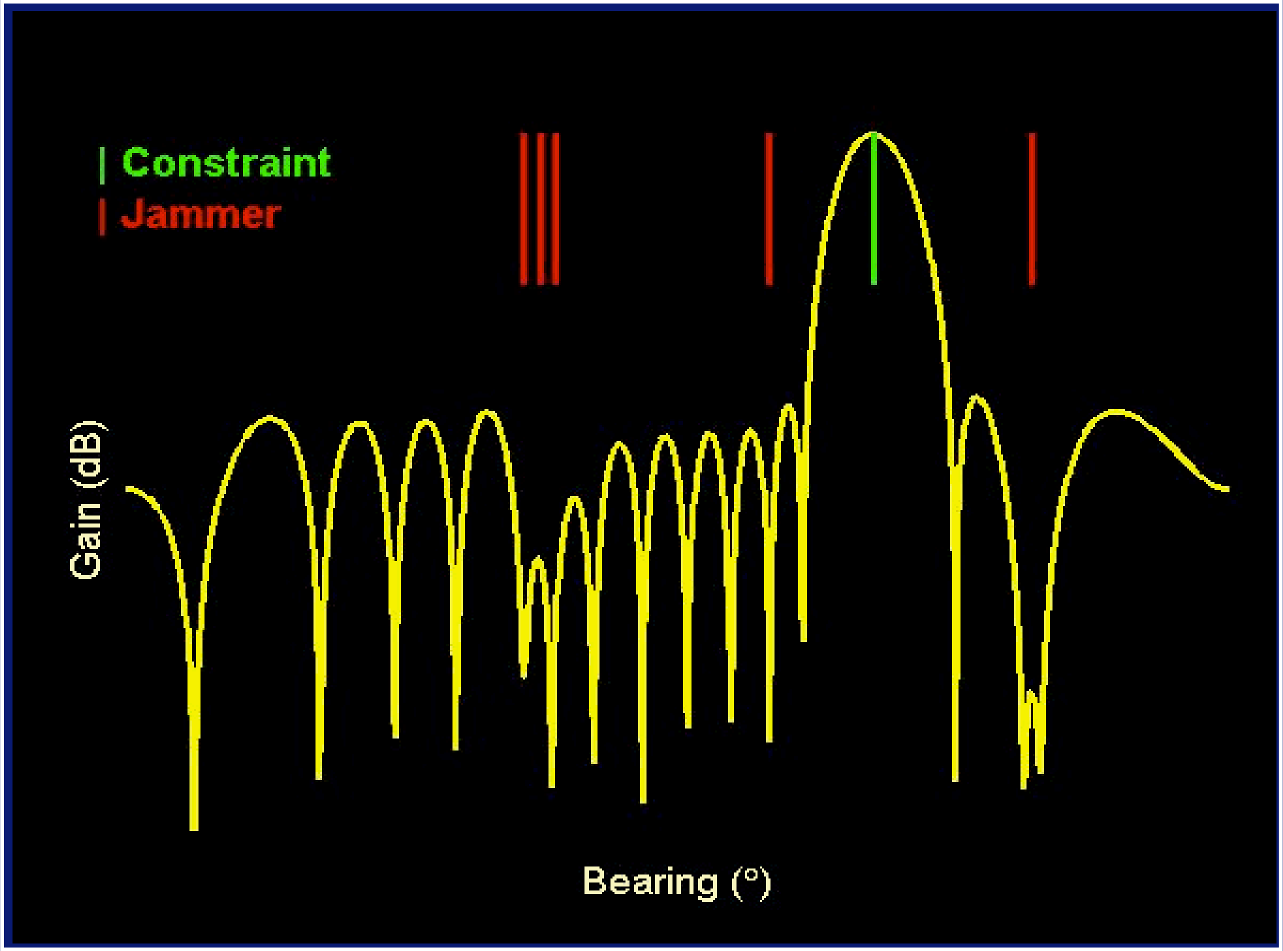
- where

$$\mathbf{Z} = \int_{-\pi/2}^{+\pi/2} h(\theta)\mathbf{c}(\theta)\mathbf{c}^H(\theta)d\theta$$

- Minimise
- Closed form solution

$$E(n) + k^2\overline{E}$$

Stabilised Beam Pattern

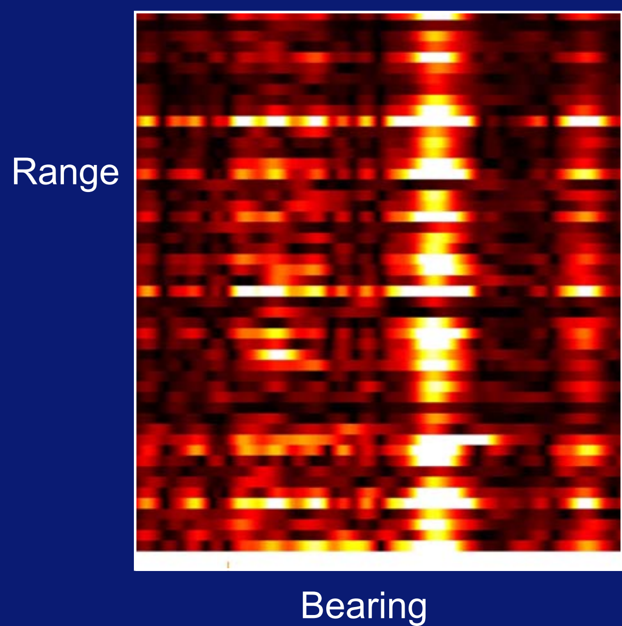


QinetiQ

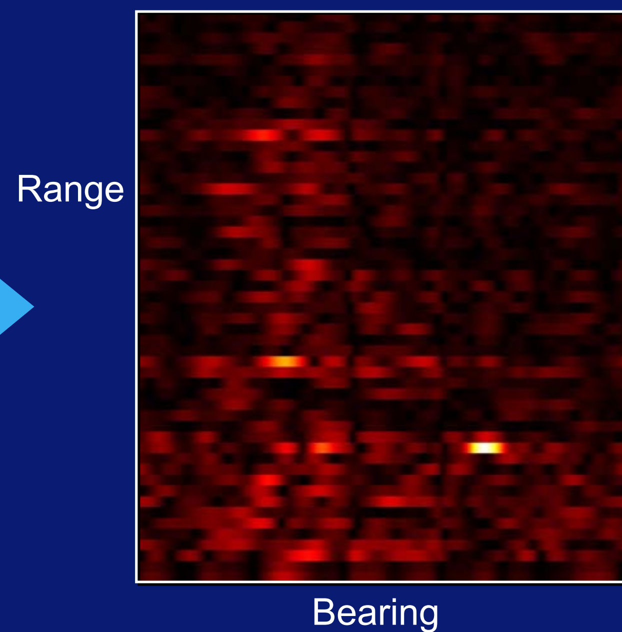
Sonobuoy Array

QinetiQ

Application to Sonar
(sonobuoy trials data)



Conventional (fixed) Beamformer



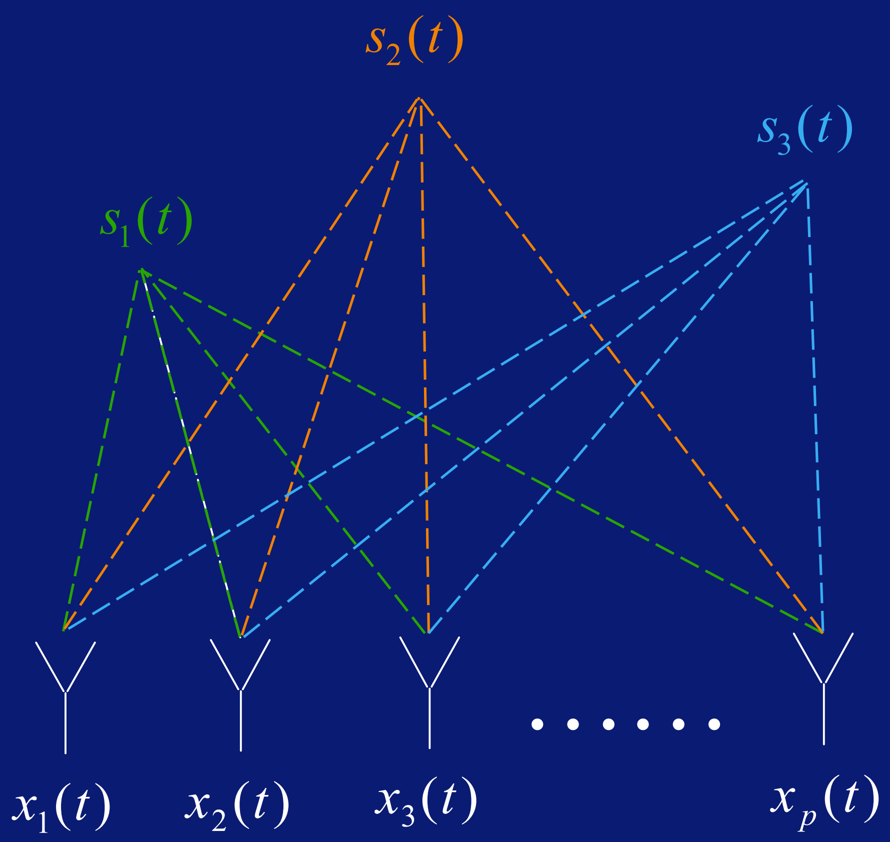
Adaptive Beamformer (stabilised)

Blind Signal Separation

- Avoids need for array calibration
  - Foetal heartbeat monitor
  - HF communications
- Independent component analysis (ICA)
- Involves use of higher order statistics (HOS)
- Requires signals to be non-Gaussian
  - Typical of man-made signals
  - Digital communication signals

QinetiQ

Blind Signal Separation



- Signal model (instantaneous)

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$$

- Data matrix

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N}$$

- Unknown mixture matrix A
- Unknown signals S
- Input signals are non-Gaussian and statistically independent

Principal Components Analysis (PCA)

- Signal model $\mathbf{X} = \mathbf{AS} + \mathbf{N}$
- Singular value decomposition (SVD)

$$\mathbf{X} = \mathbf{UDV}$$

$$= \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \mathbf{D}_s & 0 \\ 0 & \sigma\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{V}_s \\ \mathbf{V}_n \end{bmatrix}$$

$$= \mathbf{U}_s \mathbf{D}_s \mathbf{V}_s + \sigma \mathbf{U}_n \mathbf{V}_n$$

- Signal subspace $\mathbf{V}_s = \mathbf{D}_s^{-1} \mathbf{U}_s^H \mathbf{X}$

$$\mathbf{V}_s \mathbf{V}_s^H = \mathbf{I}_s$$

QinetiQ

Hidden Rotation Matrix

- By definition

$$\mathbf{V}_s \mathbf{V}_s^{H} = \mathbf{I}_s$$

- Now define

$$\tilde{\mathbf{V}}_s = \mathbf{Q}\mathbf{V}_s$$

- Then

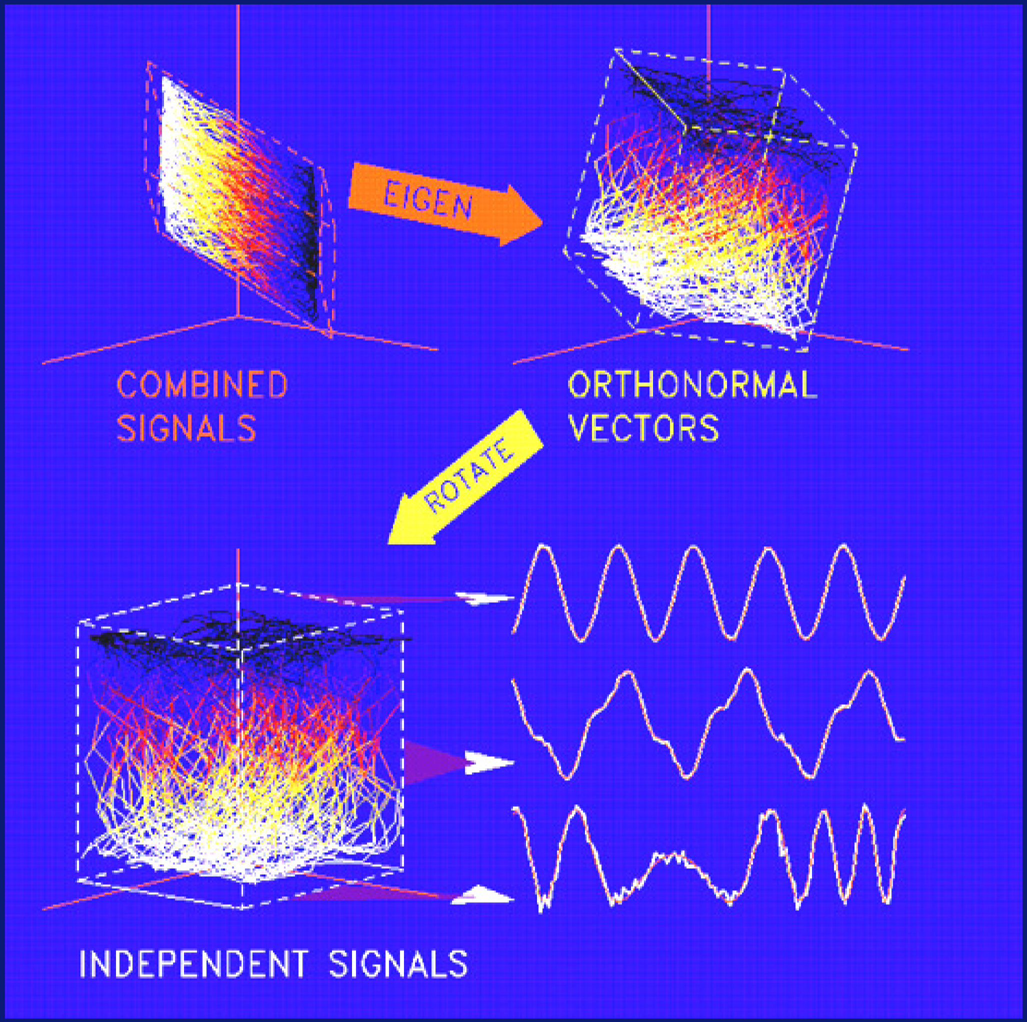
$$\tilde{\mathbf{V}}_s \tilde{\mathbf{V}}_s^{H} = \mathbf{Q}\mathbf{V}_s \mathbf{V}_s^{H} \mathbf{Q}^{H} = \mathbf{I}_s$$

- Can only conclude that

$$\mathbf{S} = \mathbf{Q}\mathbf{V}_s$$

QinetiQ

Independent Component Analysis



QinetiQ

Higher Order Statistics

- Fourth order *cumulant tensor*

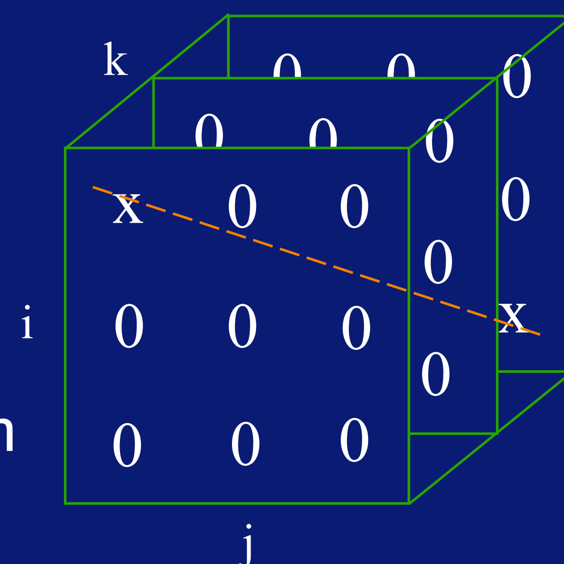
$$K_{ijkl} = \mathrm{E}\{x_i x_j x_k x_l\}$$

$$- \mathrm{E}\{x_i x_j\}\mathrm{E}\{x_k x_l\} - \mathrm{E}\{x_i x_k\}\mathrm{E}\{x_j x_l\} - \mathrm{E}\{x_i x_l\}\mathrm{E}\{x_j x_k\}$$

- Statistically independent signals

$$K_{ijkl} = k_i \quad \text{if} \quad i = j = k = l$$
$$= 0 \qquad \text{otherwise}$$

- Separation ➡ *tensor* diagonalisation
- Need for novel mathematical research



QinetiQ

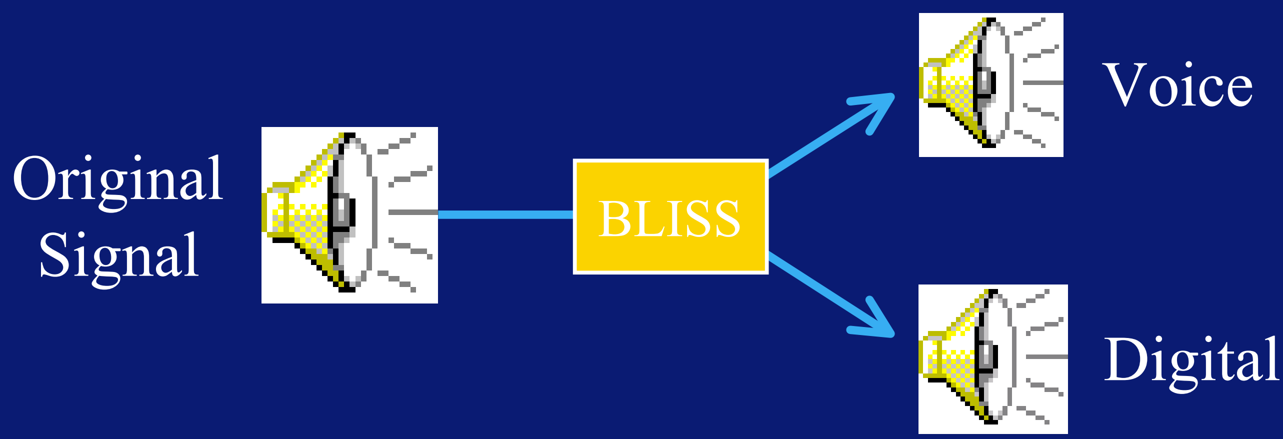
HF Communications Array

BLISS Trials Results

- HF communications data
- FSK signal 30dB stronger than SSB voice signal

TX1 Mode13454kHzSSBTX2 Mode13454kHzFSKAngularOffsetRelativelevelsSampleRateBFOFreq.Receivel

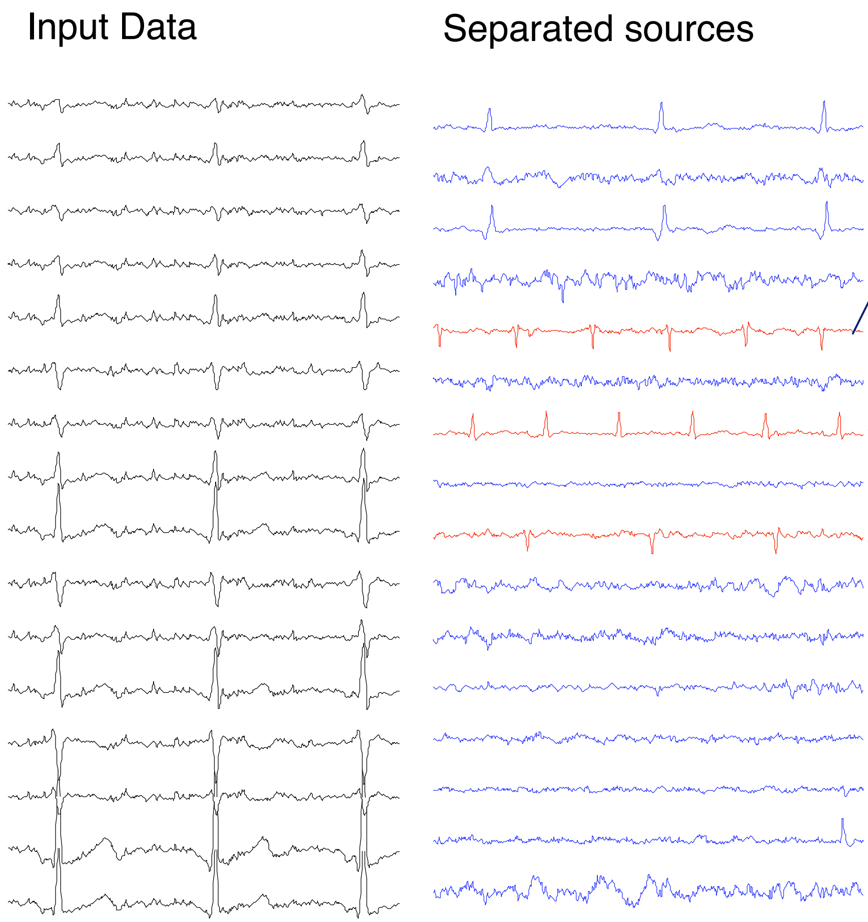
- *BLISS* algorithm - 16384 samples

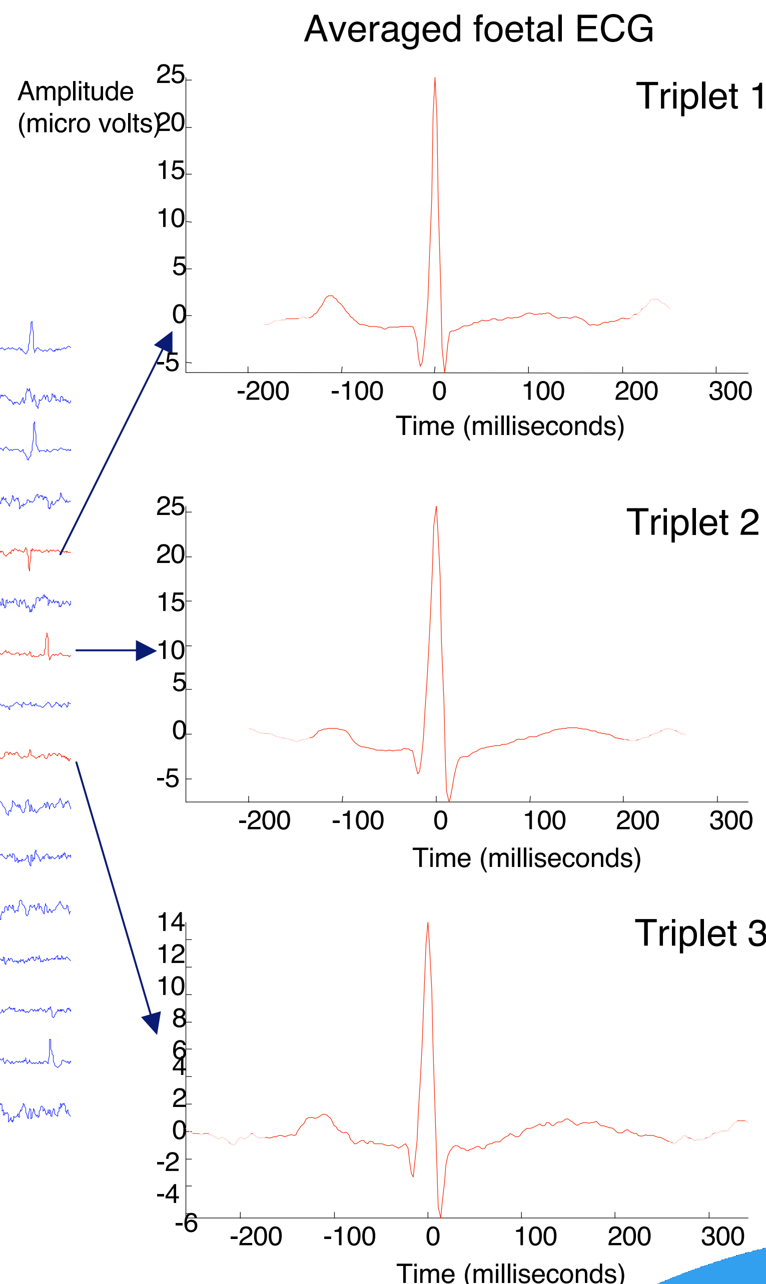


QinetiQ

Foetal Heartbeat Analysis

Application to triplets





Fast ICA (real data)

- Find unit norm vector to maximise

$$\mathrm{kurt}(\mathbf{w}^T\mathbf{x}) = \mathrm{E}\{(\mathbf{w}^T\mathbf{x})^4\} - 3\|\mathbf{w}\|^4$$

- Nonlinear adaptive filter (stochastic gradient)

$$\mathbf{w}(t+1) = \mathbf{w}(t) \pm \mu[\mathbf{x}(t)(\mathbf{w}^T(t)\mathbf{x}(t))^3 - 3\|\mathbf{w}(t)\|^2\mathbf{w}(t) + \lambda\mathbf{w}(t)]$$

- Fixed point ($t \to \infty$)

$$\mathbf{w} \propto \mathrm{E}\{\mathbf{x}(\mathbf{w}^T\mathbf{x})^3\} - 3\mathbf{w}\|\mathbf{w}\|^2$$

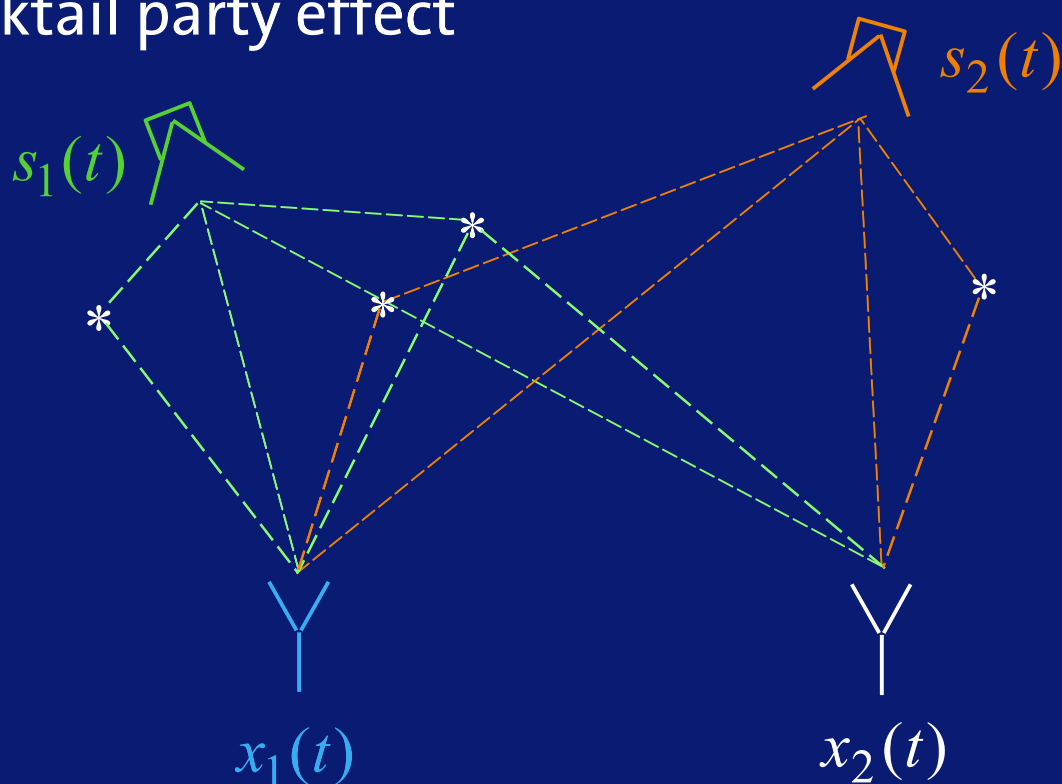
- Iterative solution (normalise and repeat)

$$\mathbf{w}(n+1) = \mathrm{E}\{\mathbf{x}(\mathbf{w}^T(n)\mathbf{x})^3\} - 3\mathbf{w}(n)$$

- Deflate/project to find next weight vector

Convolutive Mixing

- Effects of dispersion, multipath etc
  - Typical of acoustics in a room
  - Cocktail party effect



Channel Model

- Weighted sum of delayed samples (convolution)

$$x(n) = h_0 s(n) + h_1 s(n-1) + \ldots\ldots\ldots h_p s(n-p)$$

- Express in *polynomial* form (z-transform)

$$h(z) = h_0 + h_1 z^{-1} + \ldots\ldots\ldots h_p z^{-p}$$

$$s(z) = s(0) + s(1) z^{-1} + \ldots\ldots\ldots s(n) z^{-n} + \ldots\ldots\ldots$$

$$x(z) = x(0) + x(1) z^{-1} + \ldots\ldots\ldots x(n) z^{-n} + \ldots\ldots\ldots$$

- Convolution becomes *simple product*

$$x(z) = h(z) s(z)$$

QinetiQ

Polynomial Matrices

- Convolution is product of z-transforms

$$x(z) = h(z)s(z)$$

- Two signals and two sensors

$$\begin{bmatrix} x_1(z) \\ x_2(z) \end{bmatrix} = \begin{bmatrix} h_{11}(z) & h_{12}(z) \\ h_{21}(z) & h_{22}(z) \end{bmatrix} \begin{bmatrix} s_1(z) \\ s_2(z) \end{bmatrix}$$

- Polynomial matrix $\mathbf{H}(z)$
- Need for new mathematical algorithms

QinetiQ

Second Order Stage (Convolutive)

- Strong decorrelation

$$\sum_{t=1}^{T} v_i(t) v_j(t-\tau) = \sigma_i(\tau)\delta_{ij}$$

$$v_i(z) v_j(1/z) = \sigma_i(z)\delta_{ij}$$

$$\mathbf{V}(z)\mathbf{V}^T(1/z) = \begin{bmatrix} \sigma_1(z) & 0 \\ 0 & \sigma_2(z) \end{bmatrix}$$

- Whiten or equalise spectra

Hidden Paraunitary Matrix

- Paraconjugation

$$\tilde{\mathbf{H}}(z) = \mathbf{H}^T\left(1/z\right)$$

- Paraunitary matrix

$$\mathbf{H}(z)\tilde{\mathbf{H}}(z) = \tilde{\mathbf{H}}(z)\mathbf{H}(z) = \mathbf{I}$$

- Apply a decorrelation and whitening filter (2nd order)

$$\mathbf{V}(z)\tilde{\mathbf{V}}(z) = \mathbf{I}$$

- Hidden paraunitary matrix

$$\mathbf{H}(z)\mathbf{V}(z)\tilde{\mathbf{V}}(z)\tilde{\mathbf{H}}(z) = \mathbf{I}$$

QinetiQ

Future Directions

- Combine 2nd order and higher order statistics
  - semi-blind algorithms
- Combine PCA and ICA stages
  - more robust algorithms
- Broadband adaptive sensor arrays
  - broadband subspace identification

Acknowledgements

- Colleagues at QinetiQ, Malvern
  - Dr I J Clarke, Dr C A Speirs, Dr D T Hughes
  - Dr I K Proudler, Dr T J Shepherd, Mr P Baxter
- QinetiQ, Winfrith, Bincleaves, Portsdown
- University of Leuven
  - Dr L De Lathauwer
- UK Ministry of Defence
  - Corporate Research Programme

QinetiQ

QinetiQ